Robust Regression and Scale Based on Infinitesimal Neighborhoods: M- vs. AL-Estimators

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I Linear regression and scale

- Ideal Model:

$$y = x^\tau\beta + \sigma u \qquad x \sim K(dx),\ u \sim F(du) \ \text{ sto. independent}$$

- Assumptions on F and K:

(F1) F is symmetric

(F2) $\mathcal{I}_F^{\text{loc}} < \infty$ (finite Fisher information of location)
i.e., F has abs. continuous density f and
$\mathcal{I}_F^{\text{loc}} = \int (f'/f)^2 f\,d\lambda < \infty$ (Huber 81)

(F3) $\mathcal{I}_F^{\text{sc}} < \infty$ (finite Fisher information of scale)
i.e., $u \mapsto uf(u) =: uf$ abs. continuous and
$\mathcal{I}_F^{\text{sc}} = \int (1 + u(f'/f))^2 f\,d\lambda < \infty$ (Rieder, Ruckdeschel 02)

(K) $\mathrm{rk}(\mathcal{K}) = k$ with $\mathcal{K} := \int xx^\tau K(dx) \in \mathbb{R}^{k\times k}$

- Model distributions: $\theta = (\beta^\tau, \sigma)^\tau \in \mathbb{R}^k \times (0,\infty)$

$$P_\theta(dx, dy) = \frac{1}{\sigma} f\Big(\frac{y - x^\tau\beta}{\sigma}\Big)\lambda(dy)K(dx)$$

model invariant under $g_\theta(x,y) := (x, x^\tau\beta + \sigma y)$;
i.e., $P_\theta = g_\theta(P_{\theta_0})$, where $\theta_0 := (0,1)^\tau$.

- Scores:

$$\Lambda_\theta(x,y) = \frac{1}{\sigma}\begin{pmatrix} x\Lambda_f\big(\frac{y-x^\tau\beta}{\sigma}\big) \\ \frac{y-x^\tau\beta}{\sigma}\Lambda_f\big(\frac{y-x^\tau\beta}{\sigma}\big) - 1\end{pmatrix} = \frac{1}{\sigma}\Lambda_{\theta_0}\circ g_\theta^{-1}(x,y)$$

where $\Lambda_f(u) := -(f'/f)(u)$ and $u = \frac{y-x^\tau\beta}{\sigma}$.

- Fisher information: $\mathcal{I}_\theta = \frac{1}{\sigma^2}\mathcal{I}_{\theta_0}$, where by **(F1)**

$$\mathcal{I}_{\theta_0} = \begin{pmatrix} \mathcal{I}_f^{\text{loc}}\mathcal{K} & 0 \\ 0 & \mathcal{I}_f^{\text{scal}}\end{pmatrix}$$

- Proposition:
The parametric family

$$\mathcal{P} := \{P_\theta \,|\, \theta = (\beta^\tau, \sigma)^\tau \in \mathbb{R}^k \times (0,\infty)\}$$

is L_2-differentiable at θ with L_2-derivative Λ_θ.
[Swensen (80), Ruckdeschel (01), Rieder (94)]

- Influence Curves: to given θ the set of all $\eta_\theta \in L_2^{k+1}(P_\theta)$ s.t.

$$\mathrm{E}\,\eta_\theta = 0, \qquad \mathrm{E}\,\eta_\theta\Lambda_\theta^\tau = \mathbb{I}_{k+1}$$

- As. Linear Estimators: Class of all sequences of estimators S_n for θ s.t.

$$\sqrt{n}\,(S_n - \theta) = \frac{1}{n}\sum_{j=1}^n \eta_\theta((x^\tau, y)_j^\tau) + \mathrm{o}_{P_\theta^n}(n^0)$$

- Correspondence between the set of ICs at θ and the set of ICs at $\theta_0 = (0,1)^\tau$:

$$\eta_\theta(x,y) = \sigma\eta_{\theta_0}\Big(x, \frac{y - x^\tau\beta}{\sigma}\Big) = \sigma\eta_{\theta_0} \circ g_\theta^{-1}(x,y)$$

- unconditional, or errors-in-variables, $r/\sqrt{n}$-neighborhoods about P_{θ_0} at sample size n:

$$Q_n = \Big(1 - \frac{r}{\sqrt{n}}\Big)P_{\theta_0} + \frac{r}{\sqrt{n}}H$$

In the sequel, expectation is always taken under P_{θ_0}.

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Motivation

Following Huber (81, p.135) the simultaneous M-estimate of location and scale is any pair of estimators (T_n, S_n) determined by two equations of form (ψ generalizes Λ_f)

$$\textbf{(4.3)} \qquad \sum_{i=1}^n \psi\Big(\frac{u_i - T_n}{S_n}\Big) = 0$$

$$\textbf{(4.4)} \qquad \sum_{i=1}^n \Big[\frac{u_i - T_n}{S_n}\psi\Big(\frac{u_i - T_n}{S_n}\Big) - 1\Big] = 0 \qquad \textbf{(M)}$$

where he without argument generalizes **(4.4)** to his

$$\textbf{(4.6)} \qquad \sum_{i=1}^n \chi\Big(\frac{u_i - T_n}{S_n}\Big) = 0 \qquad \textbf{(AL)}$$

χ an arbitrary function, independent of ψ

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Schematic overview of the talk

	AL	M	BM
total bound	II	III	—
separate bounds	IV	V	VI

- total bound: simultaneous estimation of β and σ
- separate bounds: sequential estimation of β and σ

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II AL-estimators: total bound

- MSE problem for $r \in [0,\infty)$, $\eta \in L_2^{k+1}(P_{\theta_0})$:

$$\mathrm{E}\,|\eta|^2 + r^2 \sup{}_{P_{\theta_0}} |\eta|^2 = \min! \qquad \mathrm{E}\,\eta = 0,\ \mathrm{E}\,\eta\Lambda_{\theta_0}^\tau = \mathbb{I}_{k+1}$$

- Solution by Rieder (94) using **(F1)**:

$$\tilde\eta(x,u) = \begin{pmatrix}\tilde\eta_{\mathrm{rg}}(x,u)\\ \tilde\eta_{\mathrm{sc}}(x,u)\end{pmatrix} = \begin{pmatrix} A_{\mathrm{rg}}x\Lambda_f(u) \\ A_{\mathrm{sc}}(u\Lambda_f(u) - a_{\mathrm{sc}})\end{pmatrix} w(x,u)$$

with

$$w(x,u) = \min\Big\{1, \frac{b}{|Y|}\Big\}$$

and

$$|Y| := \big[|A_{\mathrm{rg}}x|^2\Lambda_f(u)^2 + A_{\mathrm{sc}}{}^2(u\Lambda_f(u) - a_{\mathrm{sc}})^2\big]^{1/2}$$

determine a_{sc}, A_{rg}, A_{sc}, b by the implicit equations

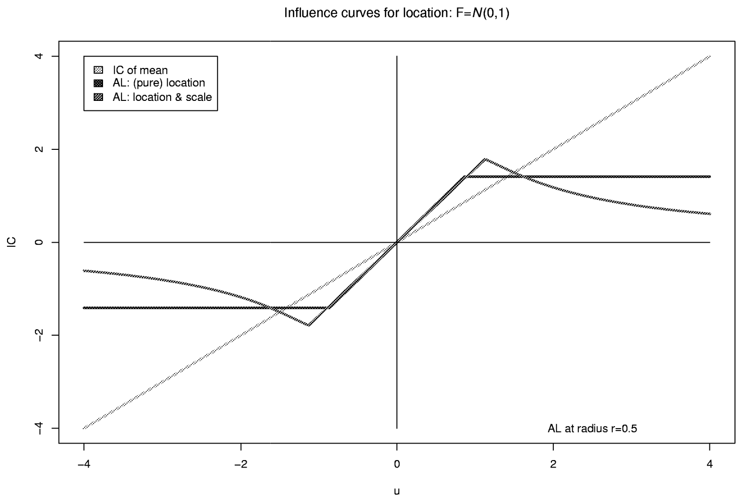
$$0 = \mathrm{E}(u\Lambda_f(u) - a_{\text{sc}})w(x,u)$$
$$\mathbb{I}_k = A_{\text{rg}}\,\mathrm{E}\,xx^\tau\Lambda_f(u)^2 w(x,u)$$
$$1 = A_{\text{sc}}\,\mathrm{E}(u\Lambda_f(u) - a_{\text{sc}})^2 w(x,u)$$

and

$$r^2 b = \mathrm{E}\left(|Y| - b\right)_+$$

- Proposition:

$$\operatorname{maxMSE}(\tilde\eta, r) = \mathrm{E}\,|\tilde\eta|^2 + r^2 \sup{}_{P_{\theta_0}} |\tilde\eta|^2 = \operatorname{tr} A_{\text{rg}} + A_{\text{sc}}$$



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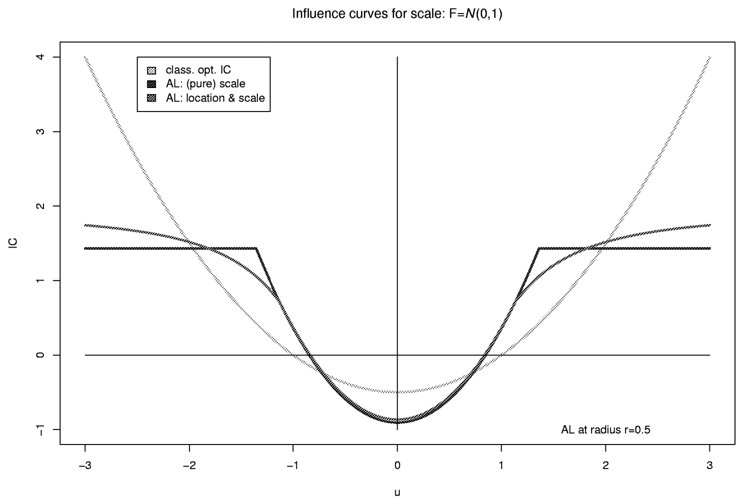

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III M-estimators: total bound

- Ideal error distribution: $\boldsymbol{F = \mathcal{N}(0,1)}$
- ICs for M-estimators: $\rho \in L_2^{k+1}(P_{\theta_0})$

$$\rho(x,u) = \begin{pmatrix} \mathcal{K}^{-1} x\psi(x,u) \\ \gamma(u\psi(x,u) - 1) \end{pmatrix} \qquad \gamma \in \mathbb{R} \text{ fixed}$$

where $\mathrm{E}\,\rho = 0$, $\mathrm{E}\,\rho\Lambda_{\theta_0}^\tau = \mathbb{I}_{k+1}$

- Sufficient conditions for $\psi \in L_2(P_{\theta_0})$: $(\mathrm{E}_\bullet = \int dF)$

(Ms1) $\psi(x,.)$ is odd in u a.e. $K(dx)$

(Ms2) $\mathrm{E}_\bullet\, u\psi(x,u) = 1$ a.e. $K(dx)$

(Ms3) $\mathrm{E}\, u^3\psi(x,u) = 1 + \gamma^{-1}$

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- MSE problem for $r \in [0,\infty)$, $\gamma \in \mathbb{R}$ fixed:

$$\mathrm{E}\,|\rho|^2 + r^2 \sup_{P_{\theta_0}} |\rho|^2 = \min! \qquad \textbf{(Ms1)-(Ms3)}$$

- Solution for fixed γ:
(by Lagrange arguments as in Rieder (94))

$$\tilde\psi(x,u) = \frac{\alpha_1(x)u + \alpha_3 u^3}{q(x,u)^2}\, w(x,u) + \gamma^2 \frac{u}{q(x,u)^2}$$

where

$$w(x,u) = \min\Big\{1, \frac{b(x,u)}{|g(x,u)|}\Big\} \qquad q(x,u)^2 = |\mathcal{K}^{-1}x|^2 + \gamma^2 u^2$$

$$g(x,u) = \frac{\alpha_1(x)u + \alpha_3 u^3}{q(x,u)} \qquad b(x,u) = \Big[b_0^2 - \gamma^2 \frac{|\mathcal{K}^{-1}x|^2}{q(x,u)^2}\Big]^{1/2}$$

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determine $\alpha_1(x)$, α_3 and b_0 by the implicit equations

$$1 = \mathrm{E}_\bullet\, u\tilde\psi(x,u)$$
$$1 + \gamma^{-1} = \mathrm{E}\, u^3\tilde\psi(x,u)$$

and

$$r^2 = \mathrm{E}\Big(\frac{|g(x,u)|}{b(x,u)} - 1\Big)_+$$

- Additionally outer optimization in γ necessary!
(done numerically)

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Results for location and scale: $F = \mathcal{N}(0,1)$

r	$\mathrm{MSE}_{\mathrm{AL},0}$	$\mathrm{MSE}_{\mathrm{M},0}$	γ_{opt}	$\mathrm{MSE}_{\mathrm{M},0}/\mathrm{MSE}_{\mathrm{AL},0}$
0.1	1.648	1.648	0.575	1.000
0.25	2.011	2.013	0.712	1.001
0.5	2.820	2.829	0.940	1.003
0.75	3.859	3.877	1.158	1.0047
0.825	4.216	4.236	1.220	1.0048
1.0	5.137	5.159	1.356	1.0043
1.25	6.679	6.693	1.510	1.002
1.5	8.506	8.508	1.575	1.000
1.75	10.637	10.641	1.581	1.000
2.0	13.092	13.096	1.584	1.000
∞	∞	∞	1.589	1.000

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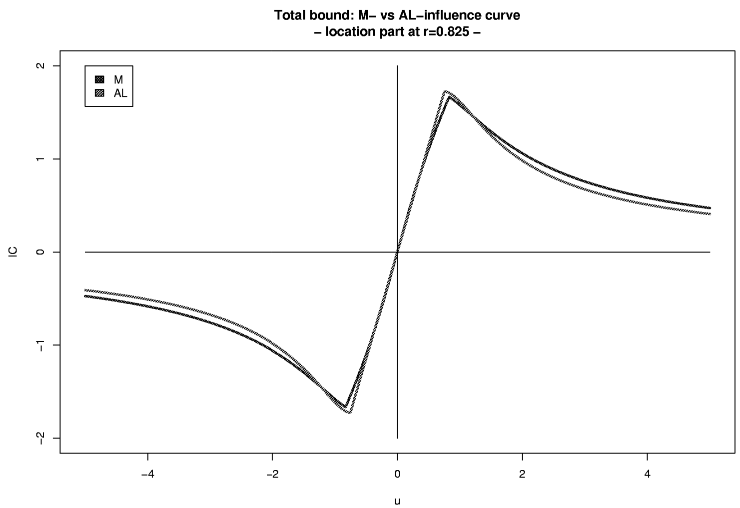

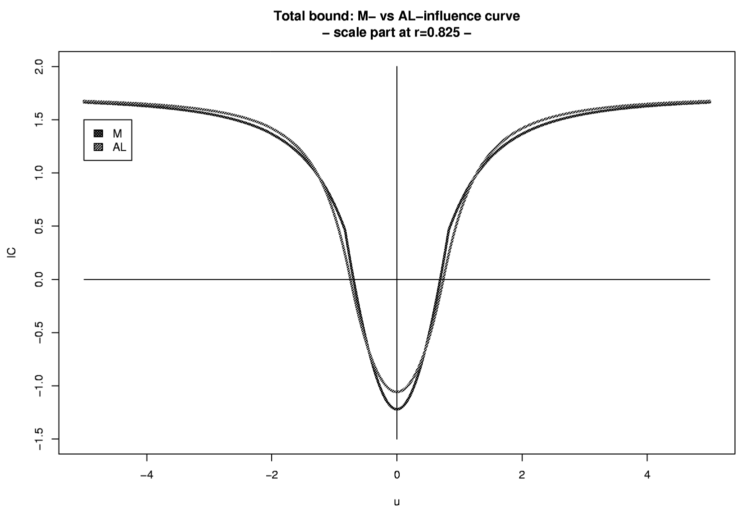

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IV AL–estimators: separate bounds

- MSE problem for $r \in [0,\infty)$, $\eta_{\rm rg} \in L_2^k(P_{\theta_0})$, $\eta_{\rm sc} \in L_2(P_{\theta_0})$:

$$\mathrm{E}\,|\eta_{\rm rg}|^2 + \mathrm{E}\,|\eta_{\rm sc}|^2 + r^2(\sup{}_{P_{\theta_0}}|\eta_{\rm rg}|^2 + \sup{}_{P_{\theta_0}}|\eta_{\rm sc}|^2) = \min!$$

under

$$\mathrm{E}\,\eta_{\rm rg} = 0 \qquad \mathrm{E}\,\eta_{\rm rg}\Lambda_f x^\tau = \mathbb{I}_k \qquad \boxed{\mathrm{E}\,\eta_{\rm rg} u\Lambda_f = 0}$$
$$\mathrm{E}\,\eta_{\rm sc} = 0 \qquad \mathrm{E}\,\eta_{\rm sc} u\Lambda_f = 1 \qquad \boxed{\mathrm{E}\,\eta_{\rm sc}\Lambda_f x^\tau = 0}$$

- MSE problem falls apart into two separate MSE problems by **robust adaptivity**, which follows by **(F1)**

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- Problem 1 **(regression part)**: $\eta_{\rm rg} \in L_2^k(P_{\theta_0})$

$$\mathrm{E}\,|\eta_{\rm rg}|^2 + r^2 \sup{}_{P_{\theta_0}}|\eta_{\rm rg}|^2 = \min!$$

under

$$\mathrm{E}\,\eta_{\rm rg} = 0 \qquad \mathrm{E}\,\eta_{\rm rg}\Lambda_f x^\tau = \mathbb{I}_k$$

- Solution for $\eta_{\rm rg}$ by Rieder (94) using **(F1)**:

$$\tilde\eta_{\rm rg}(x,u) = A_{\rm rg} x\Lambda_f(u)\min\Big\{1, \frac{b_{\rm rg}}{|A_{\rm rg}x\Lambda_f(u)|}\Big\}$$

where

$$\mathbb{I}_k = A_{\rm rg}\,\mathrm{E}\,\Lambda_f(u)^2 x x^\tau \min\Big\{1, \frac{b_{\rm rg}}{|A_{\rm rg}x\Lambda_f(u)|}\Big\}$$

and

$$r^2 b_{\rm rg} = \mathrm{E}\,\big(|A_{\rm rg}x\Lambda_f(u)| - b_{\rm rg}\big)_+$$

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- Problem 2 **(scale part)**: $\eta_{\rm sc} \in L_2(P_{\theta_0})$

$$\mathrm{E}\,|\eta_{\rm sc}|^2 + r^2 \sup{}_{P_{\theta_0}}|\eta_{\rm sc}|^2 = \min!$$

under

$$\mathrm{E}\,\eta_{\rm sc} = 0 \qquad \mathrm{E}\,\eta_{\rm sc} u\Lambda_f = 1$$

- Solution for $\eta_{\rm sc}$ by Rieder (94): $c_{\rm sc} := \frac{b_{\rm sc}}{A_{\rm sc}}$

$$\tilde\eta_{\rm sc}(u) = A_{\rm sc}(u\Lambda_f(u) - \alpha_{\rm sc}{}^2)\min\Big\{1, \frac{c_{\rm sc}}{|u\Lambda_f(u) - \alpha_{\rm sc}{}^2|}\Big\}$$

where

$$0 = \mathrm{E}(u\Lambda_f(u) - \alpha_{\rm sc}{}^2)\min\Big\{1, \frac{c_{\rm sc}}{|u\Lambda_f(u) - \alpha_{\rm sc}{}^2|}\Big\}$$
$$1 = A_{\rm sc}\,\mathrm{E}\,|u\Lambda_f(u) - \alpha_{\rm sc}{}^2|\min\big\{|u\Lambda_f(u) - \alpha_{\rm sc}{}^2|, c_{\rm sc}\big\}$$

and

$$r^2 c_{\rm sc} = \mathrm{E}\,\big(|u\Lambda_f(u) - \alpha_{\rm sc}{}^2| - c_{\rm sc}\big)_+$$

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V M-estimators: separate bounds

- Ideal error distribution: $\boldsymbol{F = \mathcal{N}(0,1)}$
- ICs for M-estimators: $\rho \in L_2^{k+1}(P_{\theta_0})$

$$\rho(x,u) = \begin{pmatrix} \mathcal{K}^{-1} x\psi(x,u) \\ \gamma(u\psi(x,u)-1) \end{pmatrix} \qquad \gamma \in \mathbb{R} \text{ fixed}$$

where $\mathrm{E}\,\rho = 0$, $\mathrm{E}\,\rho\Lambda_{\theta_0}^{\tau} = \mathbb{I}_{k+1}$

- Sufficient conditions for $\psi \in L_2(P_{\theta_0})$: ($\mathrm{E}_\bullet = \int dF$)

(Ms1) $\psi(x,.)$ is odd in u a.e. $K(dx)$

(Ms2) $\mathrm{E}_\bullet\, u\psi(x,u) = 1$ a.e. $K(dx)$

(Ms3) $\mathrm{E}\, u^3\psi(x,u) = 1 + \gamma^{-1}$

- MSE problem for $r \in [0,\infty)$:

$$\mathrm{E}\,|\rho|^2 + r^2\omega_{\mathrm{M},2}^2(\rho) = \min! \qquad \textbf{(Ms1)-(Ms3)}$$

where

$$\omega_{\mathrm{M},2}^2(\rho) = \sup{}_{P_{\theta_0}} |\mathcal{K}^{-1}x|^2\psi^2 + \gamma^2 \sup{}_{P_{\theta_0}} |u\psi - 1|^2$$

- Robust adaptive, if

$$\mathrm{E}\, xu^2\psi(x,u) = 0$$

which follows by **(F1)** and **(Ms1)**

- **But:** This problem doesn't fall apart into two separate problems, since the two components of ρ are coupled via ψ.

- Solution for fixed γ: (by Lagrange arguments)

$$\tilde\psi(x,u) = g(x,u)w(x,u) \qquad w(x,u) = \min\left\{1, \frac{b_{\mathrm{rg,sc}}(x,u)}{|g(x,u)|}\right\}$$

where

$$g(x,u) = \frac{\alpha_1(x)u + \alpha_3 u^3}{q^2(x,u)} \qquad b_{\mathrm{rg,sc}} = \min\left\{\frac{b_{\mathrm{rg}}}{|\tilde x|}, \frac{1+b_{\mathrm{sc}}}{|u|}\right\}$$

$$q(x,u) = \sqrt{|\tilde x|^2 + \gamma^2u^2} \qquad \tilde x = \mathcal{K}^{-1}x$$

determine $\alpha_1(x)$, α_3, b_{rg} and b_{sc} by the implicit equations

$$\mathrm{E}_\bullet\, u\tilde\psi = 1 \qquad \mathrm{E}\, u^3\tilde\psi = 1+\gamma^{-1}$$

$$r^2 b_{\mathrm{rg}} = \mathrm{E}\,\tfrac{1}{|\tilde x|}\,\mathrm{E}_\bullet\left[|\alpha_1(x)u + \alpha_3u^3| - \tfrac{b_{\mathrm{rg}}q^2}{|\tilde x|}\right]_+ \cdot \mathrm{I}\left\{\tfrac{b_{\mathrm{rg}}}{|\tilde x|} < \tfrac{1+b_{\mathrm{sc}}}{|u|}\right\}$$

$$r^2\gamma^2 b_{\mathrm{sc}} = \mathrm{E}\,\mathrm{E}_\bullet\left[|\alpha_1(x) + \alpha_3u^2| - \tfrac{(1+b_{\mathrm{sc}})q^2}{u^2}\right]_+ \cdot \mathrm{I}\left\{\tfrac{b_{\mathrm{rg}}}{|\tilde x|} > \tfrac{1+b_{\mathrm{sc}}}{|u|}\right\}$$

- Additionally outer optimization in γ necessary!

Results for location and scale: $\boldsymbol{F = \mathcal{N}(0,1)}$

r	$\mathrm{MSE}_{\mathrm{AL},2}$	$\mathrm{MSE}_{\mathrm{M},2}$	γ_{opt}	$\mathrm{MSE}_{\mathrm{M},2}/\mathrm{MSE}_{\mathrm{AL},2}$
0.1	1.651	1.660	0.580	1.005
0.25	2.030	2.073	0.725	1.021
0.5	2.900	3.056	0.953	1.054
0.75	4.048	4.406	1.140	1.088
1.0	5.480	6.152	1.276	1.123
1.25	7.224	8.317	1.358	1.151
1.5	9.305	10.941	1.360	1.176
1.75	11.733	14.040	1.361	1.197
2.0	14.515	17.615	1.362	1.214
∞	∞	∞	1.395	1.295

VI BM-estimators: separate bounds

- Only location and scale with $F = \mathcal{N}(0,1)$
- Bednarski and Müller (01) propose (suboptimal) estimators for location and scale by separately solving

$$\mathrm{E}\,\psi^2 = \min! \quad \text{under } \textbf{(Ms1)}, \textbf{(Ms2)},\ |\psi| \le b_{\mathrm{loc}}$$

and

$$\mathrm{E}\,|u\psi - 1|^2 = \min! \quad \text{under } \textbf{(Ms1)-(Ms3)},\ |u\psi - 1| \le b_{\mathrm{sc}}$$

- Solution by Bednarski and Müller (01):

$$\tilde{\psi} = \operatorname{sign}(u) \min\Big\{\alpha|u|, b_{\mathrm{loc}}, \frac{1+b_{\mathrm{sc}}}{|u|}\Big\}$$

$$1 = \mathrm{E}\min\big\{\alpha u^2, b_{\mathrm{loc}}|u|, (1+b_{\mathrm{sc}})\big\}$$

solves both problems, if γ in **(Ms3)** is determined by

$$\gamma = \big[\mathrm{E}\min\big\{\alpha u^4, b_{\mathrm{loc}}|u|^3, (1+b_{\mathrm{sc}})u^2\big\} - 1\big]^{-1}$$

- No outer optimization in $\gamma \Longrightarrow$ only suboptimal solution
- For computation of the MSE we provide the optimal bounds b_{loc} and b_{sc} of the M-estimators
- Generalization to regression and scale possible, if
  - the ideal regressor distribution K has finite support
  - **(Ms3)** is replaced by

(Ms3$_\bullet$) $\mathrm{E}_\bullet\, u^3\psi(x,u) = 1 + \gamma_x^{-1} \qquad$ a.e. $K(dx)$

Then: solve a location and scale problem at each design point x

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Results for location and scale: $F = \mathcal{N}(0,1)$

r	$b_{\mathrm{M},L}$	$b_{\mathrm{M},S}$	$\mathrm{MSE}_{\mathrm{M},2}$	$\mathrm{MSE}_{\mathrm{BM},2}$	γ_{M}	γ_{BM}
0.1	2.119	4.457	1.6595	1.6595	0.5796	0.5784
0.25	1.701	2.599	2.0727	2.0728	0.7248	0.7229
0.5	1.489	1.636	3.0559	3.0563	0.9534	0.9520
0.75	1.426	1.265	4.4062	4.4065	1.1402	1.1397
1.0	1.408	1.087	6.1517	6.1519	1.2758	1.2756
1.25	1.405	1.000	8.3174	8.3175	1.3582	1.3582
1.5	1.403	1.000	10.9412	10.9412	1.3598	1.3598
1.75	1.401	1.000	14.0402	14.0402	1.3610	1.3610
2.0	1.400	1.000	17.6147	17.6148	1.3618	1.3619
∞	1.360	1.000	∞	∞	1.3945	1.3945

But: confer table V-4 as for the suboptimality of M, BM with respect to AL!

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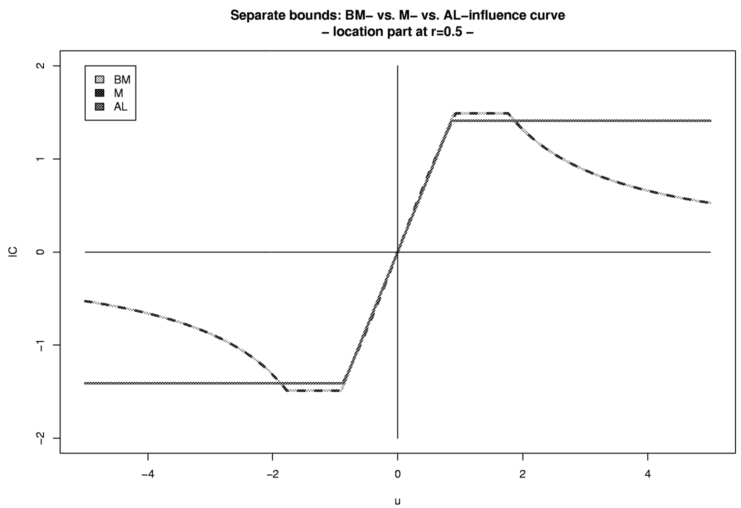

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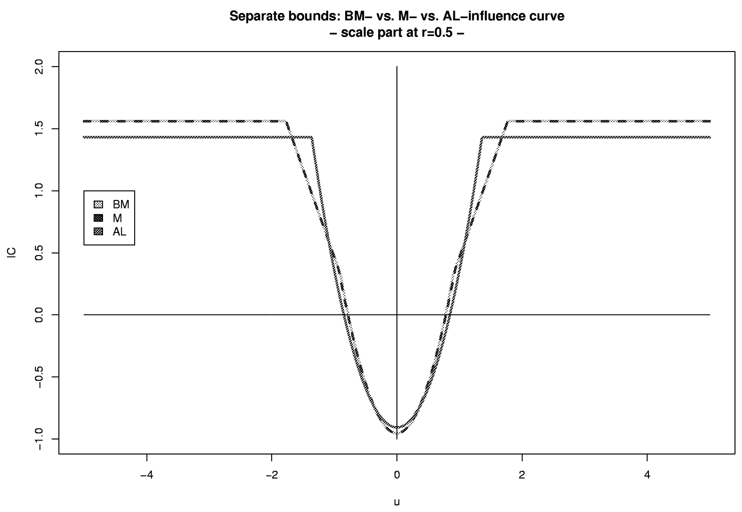

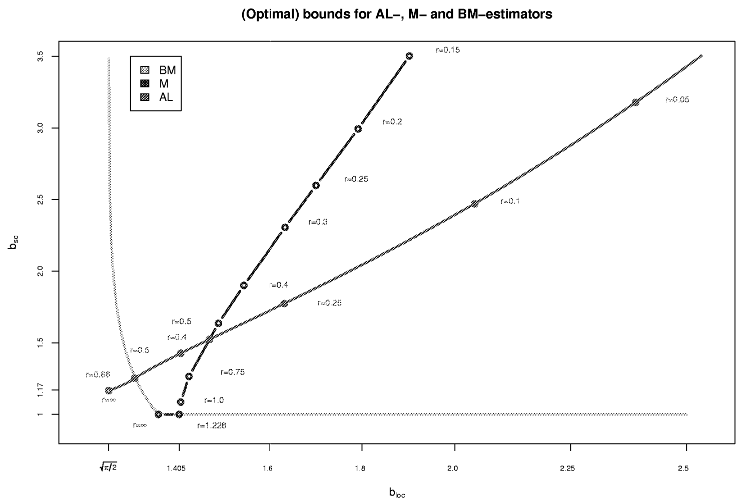

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