Numerical Contributions to the Asymptotic Theory of Robustness

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Promotionskolloquium
15.12.2005

Outline

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Ideal Model

- parametric family of probability measures

$$\mathcal{P} = \{P_\theta \,|\, \theta \in \Theta\} \qquad \Theta \subset \mathbb{R}^k \text{ (open)}$$

- smoothly parameterized; i.e., L_2 differentiable at $\theta \in \Theta$ with L_2 derivative $\Lambda_\theta \in L_2^k(P_\theta)$, $\mathrm{E}_\theta \Lambda_\theta = 0$ and

- Fisher information of full rank

$$\mathcal{I}_\theta = \mathrm{E}_\theta \Lambda_\theta \Lambda_\theta^\tau \qquad \mathcal{I}_\theta \succ 0$$

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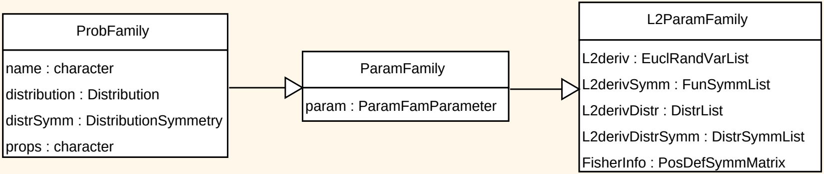
Ideal Model in S4 Classes



- meta-information slot `props`
- semi-symbolic calculus for symmetry properties
- generating functions for various L_2-families

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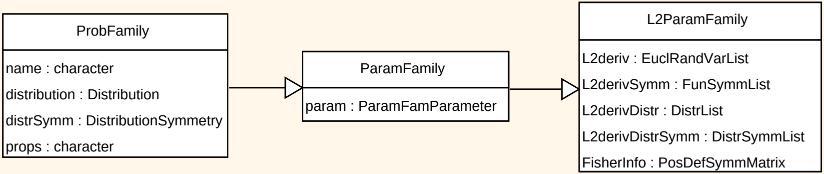

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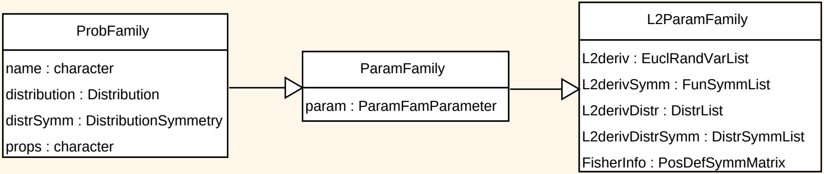

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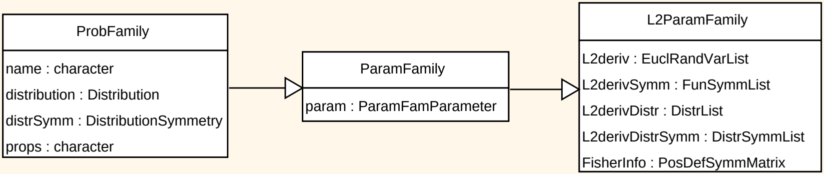

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Influence Curves (ICs) and AL Estimators

Definition

The set $\Psi_2(\theta)$ *of all* **square integrable ICs at P_θ** *is*

$$\Psi_2(\theta) = \left\{ \psi_\theta \in L_2^k(P_\theta) \,\middle|\, E_\theta\, \psi_\theta = 0,\ E_\theta\, \psi_\theta \Lambda_\theta^\tau = \mathbb{I}_k \right\}$$

Definition

An asymptotic estimator $S_n : (\Omega^n, \mathcal{A}^n) \to (\mathbb{R}^k, \mathbb{B}^k)$ *is called* **asymptotically linear at P_θ** *if there is an IC* $\psi_\theta \in \Psi_2(\theta)$ *with*

$$\sqrt{n}\,(S_n - \theta) = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} \psi_\theta(y_i) + o_{P_\theta^n}(n^0)$$

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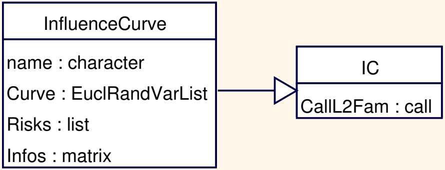

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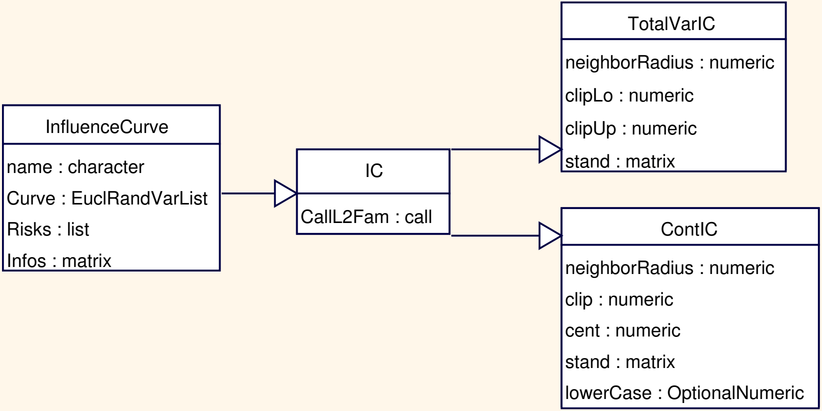

Outline

Neighborhoods

Convex contamination neighborhood of radius $r \in [0, \infty)$

$$U_c(\theta, r) = \big\{(1-r)_+ P_\theta + (1 \wedge r)\, Q \,\big|\, Q \in \mathcal{M}_1(\mathcal{A})\big\}$$

Simple perturbations for $\sqrt{n} \geq -r \inf_{P_\theta} q$, are defined as

$$dQ_n(q, r) = \left(1 + \frac{r}{\sqrt{n}} q\right) dP_\theta$$

where

$$\mathcal{G}_c(\theta) = \big\{q \in L_\infty(P_\theta) \,\big|\, E_\theta\, q = 0,\ \inf_{P_\theta} q \geq -1\big\}$$

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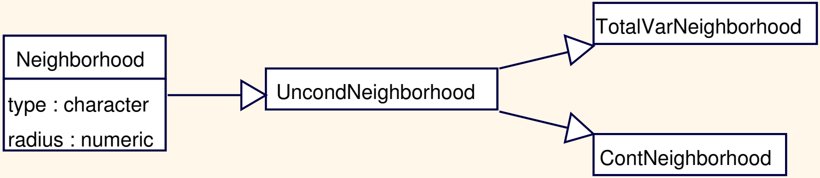

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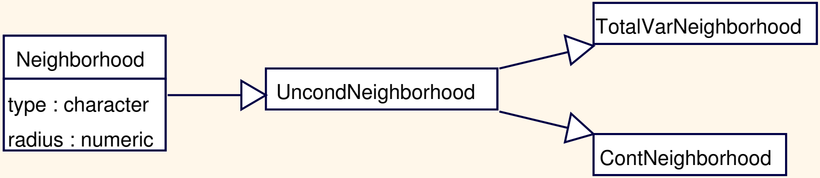

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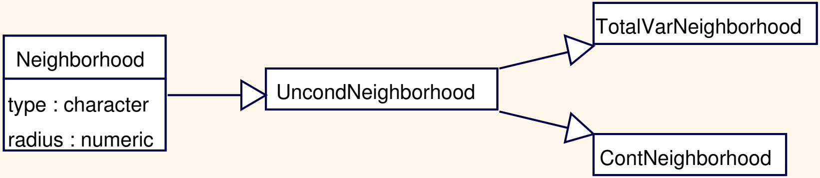

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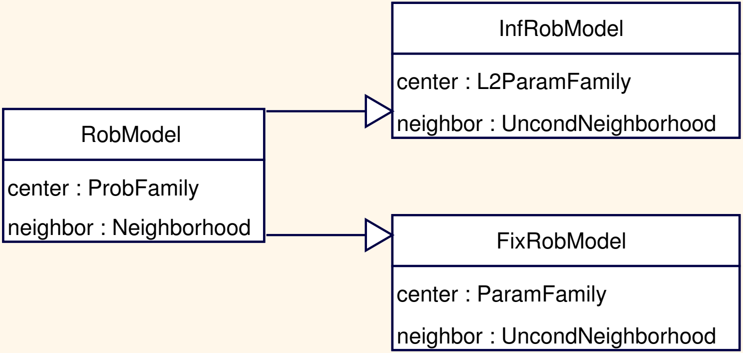
Robust Models in S4 Classes



Outline

Asymptotic Mean Square Error Problem

Choosing quadratic loss, one obtains for fixed $r \in (0, \infty)$

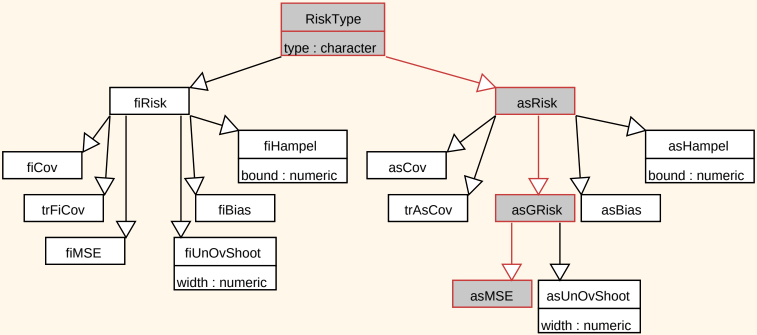
$$\mathit{maxMSE}_{\theta}(\eta_\theta, r) = E_\theta|\eta_\theta|^2 + r^2\omega_{c,\theta}(\eta_\theta)^2 = \min!$$

with $\eta_\theta \in \Psi_2^D(\theta)$ and

$$\omega_{c,\theta}(\eta_\theta) = \sup_{P_\theta} |\eta_\theta|$$

by Proposition 5.3.3 (a) of Rieder (1994).

Risks in S4 Classes



Unique MSE Solution

Theorem 5.5.7 (b), Rieder (1994)

$$\tilde{\eta}_\theta = (A_\theta \Lambda_\theta - a_\theta) w \qquad w = \min\left\{1, \frac{b_\theta}{|A_\theta \Lambda_\theta - a_\theta|}\right\}$$

with Lagrange multipliers A_θ, a_θ and b_θ determined by

$$0 = E_\theta (\Lambda_\theta - z_\theta) w \qquad a_\theta = A_\theta z_\theta$$

$$\mathbb{I}_k = A_\theta E_\theta (\Lambda_\theta - z_\theta)(\Lambda_\theta - z_\theta)^\tau w$$

$$r^2 b_\theta = E_\theta \left(|A_\theta \Lambda_\theta - a_\theta| - b_\theta\right)_+$$

Outline

Properties of MSE Solution I

- Classical Cramér-Rao bound:

$$Cov(\eta_\theta) \succeq Cov(\hat{\psi}_\theta) = \mathcal{I}_\theta^{-1} \qquad \text{where } \hat{\psi}_\theta = \mathcal{I}_\theta^{-1}\Lambda_\theta$$

Hence

$$MSE\,(\eta_\theta) = tr\, Cov(\eta_\theta) \geq tr\, Cov(\hat{\psi}_\theta) = tr\, \mathcal{I}_\theta^{-1} = MSE\,(\hat{\psi}_\theta)$$

- Generalized by Proposition 2.1.1, Kohl (2005):

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Properties of MSE Solution II

- Lagrange multipliers are bounded (Kohl (2005))
- Lagrange multipliers are not necessarily unique (Rieder (1994), Kohl (2005))
- Lagrange multipliers are continuous w.r.t. radius $r \in (0, \infty)$ (Kohl (2005))
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MSE–Inefficiency

Definition

The MSE–inefficiency *of* $\tilde{\eta}_{r_0}$ *w.r.t.* $\tilde{\eta}_r$ *is defined as*

$$relMSE(\tilde{\eta}_{r_0}, r) = \frac{maxMSE(\tilde{\eta}_{r_0}, r)}{maxMSE(\tilde{\eta}_r, r)}$$

where

$$maxMSE(\tilde{\eta}_{r_0}, r) = E\,|\tilde{\eta}_{r_0}|^2 + r^2\omega_*(\tilde{\eta}_{r_0})^2 \qquad * = c, v$$

Remark

The MSE–inefficiency was first considered and numerically evaluated in Rieder et al. (2001).

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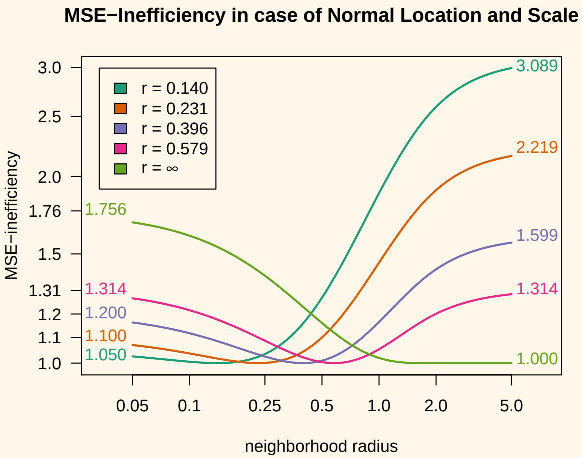
Maximum MSE–Inefficiency



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One-Step Estimator

The one-step estimator $S = (S_n)$ is defined as

$$S_n = \hat{\theta}_n + \frac{1}{n}\sum_{i=1}^{n} \psi_{n,\hat{\theta}_n(y_1,\ldots,y_n)}(y_i)$$

where $\hat{\theta}_n$ is an appropriate initial estimate.

- works in case of Exponential families of full rank (cf. Lemma 2.3.6, Kohl (2005))
- initial estimator: Kolmogorov(-Smirnov) minimum distance estimator (cf. Theorem 6.3.7, Rieder (1994))

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Optimally Robust Estimation – a Proposal

0. Choose an appropriate parametric family.
1. Choose and evaluate an appropriate initial estimate; e.g., Kolmogorov(–Smirnov) MD estimator.
2. Depending on the quality of the data, try to find a rough estimate for the amount $\varepsilon \in [0, 1]$ of gross errors such that $\varepsilon \in [\underline{\varepsilon}, \overline{\varepsilon}]$.
3. Estimate the parameter of interest by means of the corresponding radius-minimax estimator using the one-step construction.

Via our R package `ROptEst` this proposal so far works for all(!) L_2-differentiable parametric families which are based on a univariate distribution.

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Normal Location and Scale

- Ideal Model:

$$\mathcal{P} = \left\{ P_\theta = \mathcal{N}(\mu, \sigma^2) \,\middle|\, \theta = (\mu, \sigma)^\tau \in \mathbb{R} \times (0, \infty) \right\}$$

- L_2 derivative and Fisher information at $\theta = (\mu, \sigma)^\tau$:

$$\Lambda_\theta(y) = \frac{1}{\sigma} \begin{pmatrix} (y-\mu)/\sigma \\ (y-\mu)^2/\sigma^2 - 1 \end{pmatrix} \qquad \mathcal{I}_\theta = \frac{1}{\sigma^2} \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$$

- Invariant under the group of transformations

$$g_\theta(u) = \sigma u + \mu$$

i.e., $P_\theta = g_\theta(P_{\theta_0})$ where $\theta_0 = (0, 1)^\tau$

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Poisson and Normal Approximation of Binomial Distribution

- Poisson approximation: Let $\lambda = mp$ (p small, m large). Then,

$$Binom\,(m, p) \approx Pois\,(\lambda)$$

- Normal approximation: ($mp(1-p) \geq 9$)

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Setup

- Let
$$\mathcal{P}_\nu = \{P_{\nu,\theta} \,|\, \theta \in \Theta\} \subset \mathcal{M}_1(\mathcal{A}_\nu) \qquad (\nu \in \mathbb{N}_0)$$
be a sequence of L_2-differentiable parametric families.
- In addition, let $r_n := r/\sqrt{n}$ and consider
$$U_{c,\nu}(\theta, r_n) = \big\{(1 - r_n)_+ P_{\nu,\theta} + (1 \wedge r_n)\, Q_\nu \,\big|\, Q_\nu \in \mathcal{M}_1(\mathcal{A}_\nu)\big\}$$
- Question:
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Convergence of Experiments

Definition 2.2.1, Le Cam and Lo Yang (2000)

The deficiency $\delta(\mathcal{P}_\nu, \mathcal{P}_0)$ of $\mathcal{P}_\nu$ w.r.t. $\mathcal{P}_0$ is the smallest number $\delta \in [0,1]$ such that for every arbitrary loss function W with $0 \le W \le 1$ and every risk function r_2 there is an risk function r_1 such that $r_1(P_{\nu,\theta}, W) \le r_2(P_{0,\theta}, W) + \delta$ for all $\theta \in \Theta$.

Theorem 2.4.1, Kohl (2005)

Assume the laws of the corresponding L_2 derivatives as well as the trace of the corresponding Fisher information converge under suitable standardizations. Then, the suitable standardized maximum asymptotic MSE of the corresponding optimally robust estimators converges.

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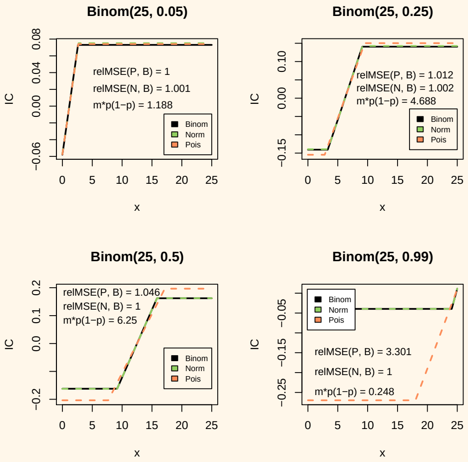
Approximation of ICs for $r = 0.25$



Bibliography

- M. Kohl.
  Numerical Contributions to the Asymptotic Theory of Robustness. Dissertation, University of Bayreuth, 2005.
- L. Le Cam and G. Lo Yang.
  Asymptotics in Statistics. Springer, 2000.
- H. Rieder.
  Robust Asymptotic Statistics. Springer, 1994.
- H. Rieder, M. Kohl and P. Ruckdeschel.
  The Costs of not Knowing the Radius. Submitted. Appeared as discussion paper Nr. 81. SFB 373, Humboldt University, Berlin, 2001.